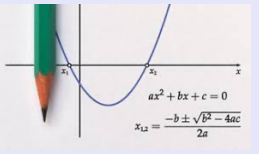
Math 125

Spring 2022

Lecture 11



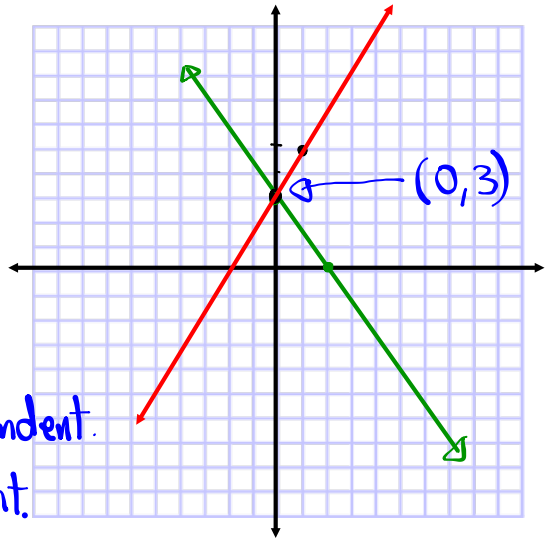
Solve by graphing

$$\begin{cases} 3x+2y=6 \\ y=2x+3 \end{cases}$$

x	y
0	3
2	0

$m = 2 = \frac{2}{1}$

Y-Int $(0,3)$



Equations are independent.

System is Consistent.

Solve by Subs. method → Isolate one variable

$$\begin{cases} 3y = 5x + 6 \\ x + y = 2 \end{cases}$$

$x = 2 - y$

$3y = 5(2 - y) + 6$

$3y = 10 - 5y + 6$

$3y + 5y = 16$

$8y = 16 \rightarrow \boxed{y = 2}$

$x = 2 - 2$

$\boxed{x = 0}$

Final Ans: $(0, 2)$

Solution Set: $\{(0, 2)\}$

Equations are independent.

System is consistent.

Solve by addition Method: Elimination Method

$$2\begin{cases} 4x + y = -23 \\ x - 2y = -17 \end{cases} \Rightarrow \begin{cases} 8x + 2y = -46 \\ x - 2y = -17 \end{cases}$$

$9x = -63$

$x = \frac{-63}{9}$ $\boxed{x = -7}$

$4x + y = -23$

$4(-7) + y = -23$

$-28 + y = -23$

$y = -23 + 28$

$\boxed{y = 5}$

Final Ans: $(-7, 5)$

Solution Set: $\{(-7, 5)\}$

Independent equations, System is Consistent

Solve

$$\begin{cases} 2x - 3y = -7 \\ 5x + 4y = 17 \end{cases} \Rightarrow \begin{cases} 8x - 12y = -28 \\ 15x + 12y = 51 \end{cases}$$

$$23x = 23$$

$$\boxed{x = 1}$$

$5(1) + 4y = 17$

$4y = 17 - 5$

$4y = 12$ $\quad \boxed{y = 3}$

Final Ans $(1, 3)$

Soln. Set $\{(1, 3)\}$

Solve

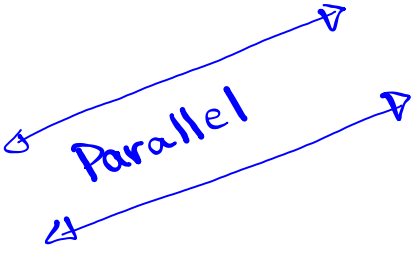
$$\begin{cases} 9x - 5y = 1 \\ -18x + 10y = 1 \end{cases} \Rightarrow \begin{cases} 18x - 10y = 2 \\ -18x + 10y = 1 \end{cases}$$

$0 = 3$

False

No Solution

$\emptyset$



Equations are independent.

System is inconsistent.

Solve

$$\begin{cases} 4x - y = 9 \\ -8x + 2y = -18 \end{cases}$$

$4x - y = 9 \Rightarrow$ Isolate $y \Rightarrow -y = -4x + 9$

$y = 4x - 9$

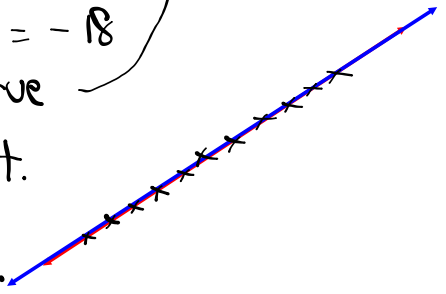
$-8x + 2(4x - 9) = -18$

$-8x + 8x - 18 = -18$

$-18 = -18$

True

Infinite # of Solutions



Equations are dependent.

System is Consistent.

Find two numbers (x & y) such that their sum is 5 and four times one plus 5 times the other one is 22.

$$\begin{cases} x + y = 5 \\ 4x + 5y = 22 \end{cases}$$

-4

$$\Rightarrow \begin{cases} -4x - 4y = -20 \\ 4x + 5y = 22 \end{cases}$$

$y = 2$

$x + 2 = 5$

$x = 3$

The numbers are 3 and 2.

S & L

Find two numbers such that their difference is 6, and the smaller one is 6 more than 5 times the larger one.

S → Smaller number

L → Larger number

$S - L = 6$

$L - S = 6$

$S = 5L + 6$

$\begin{cases} L - S = 6 \\ S = 5L + 6 \end{cases}$

$L - (5L + 6) = 6$

$L - 5L - 6 = 6$

$-4L = 12$

$L = \frac{12}{-4}$

$L = -3$

$S = 5(-3) + 6$

$= -15 + 6$

$S = -9$

Smaller number -9

Larger " -3

Find two complementary angles such that one of them is 8 times the other one.

Sum $= 90°$

$\begin{cases} x + y = 90 \\ x = 8y \end{cases}$

$8y + y = 90$

$9y = 90$

$y = 10$

$x = 8(10)$

$x = 80$

$10°$ & $80°$

OR

$80°$ & $10°$

Find two **Supplementary angles** such that one of them is 40° less than the other one.

Sum is 180°

$$\begin{cases} x + y = 180 \\ x = y - 40 \end{cases}$$

$$x = 110 - 40$$

$$x = 70$$

$$y - 40 + y = 180$$

$$2y = 220$$

$$y = 110$$

70° & 110°

A local school paid $136 for 23 tickets for a trip to the Zoo.

Adults pay $12, and children pay $5 for tickets.

How many of each?

A → # adults

C → # children

$$\begin{cases} A + C = 23 \Rightarrow A = 23 - C \\ 12A + 5C = 136 \end{cases}$$

$$12(23 - C) + 5C = 136$$

$$276 - 12C + 5C = 136$$

$$-7C = 136 - 276$$

$$-7C = -140$$

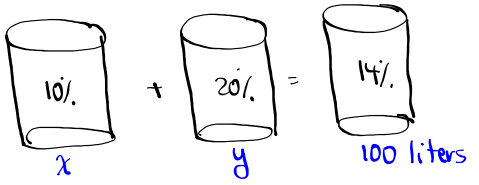
$$C = \frac{-140}{-7}$$

$$C = 20$$

20 children

3 Adults

I need 100 liters of 14% alcohol solution.
I have unlimited supply of 10% & 20% alcohol solutions. How do I mix these to obtain what I need?



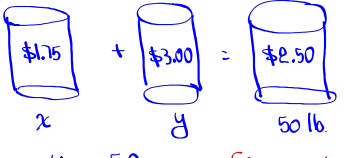
$$\begin{cases} x + y = 100 \\ 10\%\,x + 20\%\,y = 14\%\,(100) \end{cases} \Rightarrow \begin{cases} x + y = 100 \\ 10x + 20y = 14(100) \end{cases}$$

$$\begin{cases} x + y = 100 \\ x + 2y = 140 \end{cases} \Rightarrow \begin{cases} -x - y = -100 \\ x + 2y = 140 \end{cases}$$

$y = 40 \rightarrow x = 60$

60 L of 10% alcohol
&
40 L of 20% alcohol

I need 50 lb. of coffee at \$2.50/lb.
There are unlimited supply of two brands of coffee, \$1.75/lb and \$3/lb.
How do we mix these two brands to obtain what I need?



$$\begin{cases} x + y = 50 \\ 1.75x + 3.00y = 2.50(50) \end{cases} \Rightarrow \begin{cases} x + y = 50 \\ 175x + 300y = 250(50) \end{cases}$$

Divide by 5

$$\begin{cases} x + y = 50 \\ 35x + 60y = 2500 \end{cases} \quad -7 \begin{cases} x + y = 50 \\ 7x + 12y = 500 \end{cases}$$

Divide by 5 again

$$\begin{cases} -7x - 7y = -350 \\ 7x + 12y = 500 \end{cases}$$

$5y = 150$

$y = 30 \Rightarrow x = 20$

20 lb. of \$1.75/lb
&
30 lb of \$3.00/lb.

Now system of linear equations in three variables:

$$\begin{cases} x + y + z = 2 \\ 2x + y - z = 2 \\ 3x + 9y + 6z = 3 \end{cases}$$

Solution is ordered-triple (x, y, z)

Solution has to satisfy all equations

Is $(2, -1, 1)$ a solution of this system?

$x + y + z = 2$ ✓ $\quad 2x + y - z = 2$ ✓ $\quad 3x + 9y + 6z = 3$ ✓

$2 - 1 + 1 = 2$ $\quad 2(2) - 1 - 1 = 2$ $\quad 3(2) + 9(-1) + 6(1) = 3$

$2 = 2$ $\quad 4 - 2 = 2$ $\quad 6 - 9 + 6 = 3$

$-3 + 6 = 3$

Yes

Solve

$$\begin{cases} x + y + z = 2 \\ 2x + y - z = 2 \\ 3x + 9y + 6z = 3 \end{cases}$$

$$\begin{cases} x + y + z = 2 \\ 2x + y - z = 2 \end{cases}$$

$3x + 2y = 4$

$$6\begin{cases} 2x + y - z = 2 \\ 3x + 9y + 6z = 3 \end{cases} \Rightarrow \begin{cases} 12x + 6y - 6z = 12 \\ 3x + 9y + 6z = 3 \end{cases}$$

$15x + 15y = 15$

Divide by 15

$x + y = 1$

Solve

$$\begin{matrix} 3x + 2y = 4 \\ -2\{ x + y = 1 \end{matrix} \Rightarrow \begin{cases} 3x + 2y = 4 \\ -2x - 2y = -2 \end{cases}$$

$x = 2$

$2 + y = 1$

$y = -1$

$x + y + z = 2$

$2 - 1 + z = 2$

$z = 1$

Final Ans: $(2, -1, 1)$

Solution Set: $\{(2, -1, 1)\}$

Solve $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

$\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \end{cases}$

$x + 2z = 10$

$\begin{cases} 2x + y + z = 9 \\ 3x - y + z = 9 \end{cases}$

$5x + 2z = 18$

$-1 \begin{cases} x + 2z = 10 \\ 5x + 2z = 18 \end{cases} \Rightarrow \begin{cases} -x - 2z = -10 \\ 5x + 2z = 18 \end{cases}$

$4x = 8$

$\boxed{x = 2}$

$2 + 2z = 10$

$2z = 8$

$\boxed{z = 4}$

$2x + y + z = 9$

$2(2) + y + 4 = 9$

$y + 8 = 9$

$\boxed{y = 1}$

Final Ans: $(2, 1, 4)$

Solution Set $\{(2, 1, 4)\}$

Solve

$\begin{cases} x + 4y - z = 6 \\ 2x - y + z = 3 \\ 3x + 2y + 3z = 16 \end{cases}$

$3 \begin{cases} x + 4y - z = 6 \\ 3x + 2y + 3z = 16 \end{cases}$

$6x + 14y = 34$

Divide by 2

$3x + 7y = 17$

$\begin{cases} x + 4y - z = 6 \\ 2x - y + z = 3 \end{cases}$

$3x + 3y = 9$

Divide by 3

$x + y = 3$

Now Solve

$-3 \begin{cases} x + y = 3 \\ 3x + 7y = 17 \end{cases}$

$4y = 8$

$\boxed{y = 2}$

$x + 2 = 3$

$\boxed{x = 1}$

$x + 4y - z = 6$

$1 + 4(2) - z = 6$

$1 + 8 - z = 6$

$9 - z = 6$

$\boxed{z = 3}$

Final Ans $(1, 2, 3)$

Solution Set $\{(1, 2, 3)\}$

Class QZ 9

Solve

$$\begin{cases} 3x + 2y = 11 \\ 2\{ x - y = -3 \end{cases} \Rightarrow \begin{cases} 3x + 2y = 11 \\ 2x - 2y = -6 \end{cases}$$

$$5x = 5$$

$1 - y = -3$

$-y = -3 - 1$

$-y = -4$

$\boxed{x = 1}$

$\boxed{y = 4}$

$\Rightarrow \boxed{(1,4)}$

$\{(1,4)\}$